CBSE Sample Paper -03
SUMMATIVE ASSESSMENT –I
Class – X Mathematics

Time allowed: 3 hours | **ANSWERS** | Maximum Marks: 90

SECTION – A

1. $\frac{6}{1250}=\frac{3}{625}=\frac{3}{5^4}=\frac{2^4}{2^4}=\frac{48}{(5\times 2)^4}=\frac{48}{10^4}=0.0048$

This representation will terminate after 4 decimal places.

2. We have AC = BC and $AB^2 = 2AC^2$

Now, $AB^2 = 2AC^2$

$\Rightarrow \quad AB^2 = AC^2 + AC^2$

$\Rightarrow \quad AB^2 = AC^2 + BC^2$ [$\because$ AC = BC (Given)]

$\Rightarrow \quad \Delta ABC$ is a right triangle right angled at C.

3. $\sin A+\sin^2 A=1 \Rightarrow \sin a=1-\sin^2 a=\cos^4 A=1$

$\therefore \quad \cos^2 a+\cos^4 a=\sin A+\sin^4 A=1.$

4. $\cot B=\cot(90^o-A)$ $\quad (\because A+B=90^o)$

$=\tan A$ $\quad (\because \cot(90^o-\theta)=\tan\theta)$

$=\frac{3}{4}$

SECTION – B

5. We observe that $y = ax^2 + bx + c$ represents a parabola opening downwards. Therefore, $a< 0$.

We also observe that the vertex of the parabola is in first quadrant.

$\therefore -\frac{b}{2a}>0 \Rightarrow -b<0 \Rightarrow b>0$

Parabola $y = ax^2 + bx + c$ cuts Y-axis at P. On Y-axis, we have $x = 0$.

Putting $x = 0$ in $y = ax^2 + bx + c$, we get $y = c$.

So, the coordinates of P are $(0, c)$. As P lies on the positive direction of Y-axis, therefore, $c> 0$.

Hence, $a< 0$, $b> 0$ and $c> 0$.

6.

Calculation of arithmetic mean

Weight (Kg) x_i	Frequency f_i	$f_i x_i$

67	4	268
70	3	210
72	2	144
73	2	146
75	1	75
	$N = \sum f_i = 12$	$\sum f_i x_i = 843$

∴ Mean = $\overline{X} = \frac{\sum f_i x_i}{N} = \frac{843}{12} = 70.25$

7. In triangles ABE and CFB, we have

$\angle AEB = \angle CBF$ [Alternate angles]

$\angle A = \angle C$ [Opposite angles of a parallelogram]

Thus, by AA-criterion of similarity, we have

$\Delta ABE \sim \Delta CFB$

8. In ΔABC, we have

$AB = 20$, $\angle BAC = 60°$

∴ $\tan\angle BAC = \frac{BC}{AB}$

⇒ $\tan 60° = \frac{BC}{20}$

⇒ $\sqrt{3} = \frac{BC}{20}$ ⇒ $BC = 20\sqrt{3}$ cm

9. Required number of minutes is the LCM of 18 and 12.

We have,

$18 = 2 \times 3^2$ and $12 = 2^2 \times 3$

∴ LCM of 18 and 12 = $2^2 \times 3^2 = 36$

Thus, Prenu and Raj will meet again at the starting point after 36 minutes.

10. If α, β and γ are the zeros of a cubic polynomial $f(x)$, then

$f(x) = k\{x^3 - (\alpha+\beta+\gamma)x^2 + (\alpha\beta+\beta\gamma+\gamma\alpha)x - \alpha\beta\gamma\}$,

where k is any non-zero real number.

Here, $\alpha+\beta+\gamma = 2$, $\alpha\beta+\beta\gamma+\gamma\alpha = -7$ and $\alpha\beta\gamma = -14$

∴ $f(x) = k(x^3 - 2x^2 - 7x + 14)$, where k is any non-zero real number.

SECTION – C

11. We know that an old positive integer n is of the form (4q+1) or (4q+3) for some integer q.

Case I When n=(4q+1)

In this case

$n^2-1=(4q+1)^2-1=16q^2+8q=8q(2q+1)$

Which is clearly divisible by 8.

Case II When n=(4q+3)

In this case, We have

$n^2-1=(4q+3)^2-1=16q^2+14q+8=8(2q^2+3q+1)$

Which is clearly divisible by 8.

12. We have find $\cos^2 A$ in term of m and n. This means that the angle B is to be eliminated from the given relations.

Now, tan A = n tan B

$\tan B=\frac{1}{n}\tan A=\cot B=\frac{n}{\tan A}$

and sinA = m sinB

$\sin B=\frac{1}{m}\sin A= \cos ecB=\frac{m}{\sin A}$

Substituting the values of cot B and cosec B in $\cos ec^2 B-\cot^2 B=1$, we get

$\frac{m^2}{\sin^2 A}-\frac{n^2}{\tan^2 A}=1$

$\frac{m^2}{\sin^2 A}-\frac{n^2\cos^2 A}{\sin^2 A}=1$

$\frac{m^2-n^2\cos^2 A}{\sin^2 A}=1$

$m^2-n^2\cos^2 A=\sin^2 A$

$m^2-n^2\cos^2 A=1-\cos^2 A$

$m^2-1=n^2\cos^2-\cos^2 A$

$m^2-1=\left(n^2-1\right)\cos^2 A$

$\frac{m^2-1}{n^2-1}=\cos^2 A$

13. We have given $\tan\theta+\sin\theta=m$, and $\tan\theta-\sin\theta=n$, then

L.H.S = $\left(m^2 - n^2\right) = \left(\tan\theta + \sin\theta\right)^2 - \left(\tan\theta - \sin\theta\right)^2$

$\tan^2\theta + \sin^2\theta + 2\tan\theta\sin\theta - \tan^2\theta - \sin^2\theta + 2\tan\theta\sin\theta$

$= 4\tan\theta\sin\theta = 4\sqrt{\tan^2\theta\sin^2\theta}$

$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta}\left(1-\cos^2\theta\right)}$

$= 4\sqrt{\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta}$

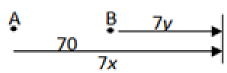
$= 4\sqrt{\tan^2\theta - \sin^2\theta}$

$= 4\sqrt{\left(\tan\theta - \sin\theta\right)\left(\tan\theta + \sin\theta\right)}$

$= 4\sqrt{mn}$ = R.H.S

14. Let the speed of faster car at A = x km/hr and the speed of slower car at B = y km.hr.

Case 1: When they travel in same direction



Distance covered by faster car in 7 hours = $7x$ km

Distance covered by slower car in 7 hours = $7y$ km

$\Rightarrow \quad 7x = 7y + 70$

$\Rightarrow \quad 7(x - y) = 70$

$\Rightarrow \quad x - y = 10$ ...(i)

Case 2: When they travel in opposite direction

Distance travelled by faster car in 1 hour = x km

Distance travelled by slower car in 1 hour = y km

$\Rightarrow \quad x + y = 70$ ...(ii)

Adding (i) and (ii), we get

$2x = 80 \quad \Rightarrow \quad x = 40$

Substituting $x = 40$ in (i), we get

$40 - y = 10 \quad \Rightarrow \quad y = 40 - 10 = 30$

$\therefore$ Speeds of cars would be 40 km/hr and 30 km/hr.

15. $x^3 - 3x^2 + x + 1$ is a cubic polynomial.

$\therefore$ Sum of its zeros = $\frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3} = \frac{-(-3)}{1} = 3$

$\Rightarrow$ $a - b + a + a + b = 3$

$\Rightarrow$ $3a = 3$

$\Rightarrow$ $a = 3$

Also, product of its zeros = $\frac{-(\text{constant term})}{\text{coefficient of } x^3} = \frac{-1}{1} = -1$

$\Rightarrow$ $(a - b) \times a \times (a + b) = -1$

$\Rightarrow$ $a(a^2 - b^2) = -1$

$\Rightarrow$ $1(1 - b^2) = -1$ $[\because a = 1]$

$\Rightarrow$ $1 - b^2 = -1$

$\Rightarrow$ $b^2 = 2$

$\Rightarrow$ $b = \pm\sqrt{2}$

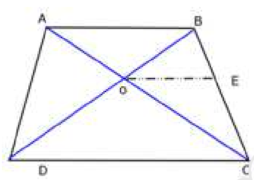
Thus, $a = 1$ and $b = \pm\sqrt{2}$.

16. Given: A quadrilateral ABCD whose diagonals AC and BC intersect at O such that

$\frac{AO}{OC} = \frac{BO}{OD}$

To prove: ABCD is a trapezium.

Construction: Through O, draw OE || AB.



$\therefore$ By Basic Proportionality Theorem, we have

$\frac{AO}{OC} = \frac{BE}{EC}$

But, $\frac{AO}{OC} = \frac{BO}{OD}$ [Given]

$\therefore$ $\frac{BE}{EC} = \frac{BO}{OD}$

Now, in ΔBCD, we have $\frac{BE}{EC} = \frac{BO}{OD}$

$\therefore$ By Basic Proportionality Theorem, we have

OE || DC

Now, OE || AB [By construction]

and, OE || DC

$\therefore$ AB || DC

Thus, ABCD is a trapezium.

17. Since, $\Delta FEC \cong \Delta GDB$

$\Rightarrow EC = BD$ ...(i)

It is given that

$\angle 1 = \angle 2$

$\Rightarrow AE = AD$ [Sides opposite to equal angles are equal] ...(ii)

From (i) and (ii)

$$\frac{AE}{EC} = \frac{AD}{BD}$$

$\Rightarrow DE \| BC$ [By the converse of basic proportionality theorem]

$\Rightarrow \angle 1 = \angle 2 \ and \ \angle 2 = \angle 4$ [Corresponding]

Thus, in $\Delta 's$ ADE and ABC, we have

$\angle A = \angle A$ $[Common]$

$\angle 1 = \angle 3$

$\angle 2 = \angle 4$ $[\text{Proved above}]$

So, by AAA criterion of similarity, we have

$\Delta ADE \sim \Delta ABC$

18. We have,

$$\frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}} \Rightarrow \frac{\frac{\cos\theta - \sin\theta}{\cos\theta}}{\frac{\cos\theta + \sin\theta}{\cos\theta}} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

[Dividing numerator & denominator of the LHS by $\cos\theta$]

$$\Rightarrow \frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\sqrt{3}}{1+\sqrt{3}}$$

On comparing we get,

$\Rightarrow \tan\theta = \sqrt{3} \ \Rightarrow \ \tan\theta = \tan 60^o \quad \Rightarrow \theta = 60^o$

19. Here, the maximum class frequency is 61 and the class corresponding to this frequency is 60-80. So, the model class is 60-80.

Here, $l = 60, h = 20, f_1 = 52, f_0 = 38$

$$\therefore\ Model = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$

$$= 60 + \frac{31-52}{2\times 61 - 52 - 38} \times 20 = 60 + \frac{9}{122-90} \times 20$$

$$= 60 + \frac{9}{32} \times 20 = 60 + \frac{45}{8}$$

$$= 60 + 5.625 = 65.625$$

Hence, model lifetime of the components is 65.625 hours.

20. Let the tens and units digits of the required number be x and y, respectively. Then $xy = 14$.

Required number = $(10x + y)$

Number obtained on reversing its digits = $(10y + x)$

$\therefore \quad (10x + y) + 45 = (10y + x)$

$\Rightarrow \quad 9(y - x) = 45$

$\Rightarrow \quad y - x = 45$ ...(i)

Now, $(y+x)^2 - (y-x)^2 = 4xy$

$\Rightarrow \quad (y+x) = \sqrt{(y-x)^2 + 4xy}$

$= \sqrt{25 + 4\times 14} = \sqrt{81}$

$\Rightarrow \quad y + x = 9$ ...(ii)

On adding (i) and (ii), we get

$2y = 14 \quad \Rightarrow \quad y = 7$

Putting $y = 7$ in (ii), we get

$7 + x = 9 \quad \Rightarrow \quad x = 9 - 7 = 2$

$\therefore \quad x = 2$ and $y = 9$

SECTION – D

21. Given, $a + bk^{\frac{1}{3}} + ck^{\frac{2}{3}} = 0,$ ... (i)

Multiplying both sides by $k^{\frac{1}{3}}$, we have

$bk^{\frac{1}{3}} + ck^{\frac{2}{3}} + ck = 0,$ …(ii)

Multiplying (i) by b and (ii) by c and then subtracting , we have

$(ab+b^2k^{1/2}+bck^{2/3})-ack^{1/3}+bck^{2/3}+c^2k)=0$

$\Rightarrow (b^2-ac)k^{1/3}+ab-c^2k=0$

$\Rightarrow b^2-ac=0$ and $ab-c^2k=0$ [Since $k^{1/3}$ is irrational]

$\Rightarrow b^2-ac=0$ and $ab=c^2k$

$\Rightarrow b^2=ac$ and $a^2b^2=c^4k^2$

$\Rightarrow a^2(ac)=c^4k^2$ [By putting $b^2=ac$ in $a^2b^2=c^4k^2$]

$\Rightarrow a^3c-k^2c^4=0 \Rightarrow (a^3-k^2c^3)c=0$

$\Rightarrow a^3-k^2c^3=0$, or $c=o$

Now, $a^3-k^2c^3=0$

$\Rightarrow k^2=\dfrac{a^3}{c^3} \Rightarrow (k^2)^{1/3}=\left(\dfrac{a^3}{c^3}\right) \Rightarrow k^{2/3}=\dfrac{a}{c}$

This is impossible as $k^{2/3}$ is irrational and $\dfrac{a}{c}$ is rational.

$\therefore \quad a^3-k^2c^3\neq 0$

Hence, c=0

Substituting c =0 in $b^2 - ac = 0$, we get b = 0

Substituting b = 0 and c = 0 in $a+bk^{1/3}+ck^{2/3}=0$, we get a = 0

Hence, c = b = c = 0.

22. Let $p(x) = x^3-3x^2+x+2$

$q(x) = x - 2$

$r(x) = -2x + 4$

by division algorithm,

$p(x) = g(x) \times q(x) + r(x)$

$\Rightarrow \quad g(x) \times q(x) = p(x) - r(x)$

$\Rightarrow \quad g(x)(x - 2) = x^3-3x^2+x+2-(-2x+4)$

$= x^3-3x^2+x+2+2x-4$

$= x^3-3x^2+3x-2$

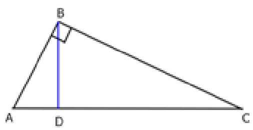
$\Rightarrow \quad g(x)$ is a factor of x^3-3x^2+3x-2 other than $(x - 2)$.

Dividing x^3-3x^2+3x-2 by $(x - 2)$, we obtain $g(x)$ as follows:

$$
\begin{array}{r|l}
 & x^2 - x + 1 \\
x - 2 & x^3 - 3x^2 + 3x - 2 \\
 & x^3 - 2x^2 \\
 & - \quad + \\ \hline
 & -x^2 + 3x - 2 \\
 & -x^2 + 2x \\
 & + \quad - \\ \hline
 & x - 2 \\
 & x - 2 \\
 & - \quad + \\ \hline
 & 0
\end{array}
$$

∴ $g(x) = x^2 - x + 1$

23. Theorem: In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given: A right-angled triangle ABC in which $\angle B = 90°$.

To prove: $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

i.e., $AC^2 = AB^2 + BC^2$

Construction: From B, draw $BD \perp AC$.

Proof: In triangles ADB and ABC, we have

$\angle ADB = \angle ABC$ [Each equal to 90°]

and, $\angle A = \angle A$ [Common]

So, by AA-similarity criterion, we have

$\Delta ADB \sim \Delta ABC$

$\Rightarrow \frac{AD}{AB} = \frac{AB}{AC}$ [∵ In similar triangles corresponding sides are proportional]

$\Rightarrow AB^2 = AD \times AC$ ...(i)

In triangles BDC and ABC, we have

$\angle CDB = \angle ABC$ [Each equal to 90°]

and, $\angle C = \angle C$ [Common]

So, by AA-similarity criterion, we have

$\Delta BDC \sim \Delta ABC$

$\Rightarrow \frac{DC}{BC} = \frac{BC}{AC}$ [∵ In similar triangles corresponding sides are proportional]

$\Rightarrow \quad BC^2 = AC \times DC$ ...(ii)

Adding equations (i) and (ii), we get

$AB^2 + BC^2 = AD \times AC + AC \times DC$

$\Rightarrow \quad AB^2 + BC^2 = AC(AD + DC)$

$\Rightarrow \quad AB^2 + BC^2 = AC \times AC$

$\Rightarrow \quad AB^2 + BC^2 = AC^2$

Or, $\quad AC^2 = AB^2 + BC^2$

24. Let the number of students be x and the number of rows be y.

Then, number of students in each row = $\frac{x}{y}$

When one student is extra in each row, there are 2 rows less, i.e., when each row has $\left(\frac{x}{y}+1\right)$ students, the number of rows is $(y-2)$.

$\therefore \quad$ Total number of students = No. of rows × No. of students in each row

$\Rightarrow \quad x = \left(\frac{x}{y}+1\right)(y-2)$

$\Rightarrow \quad x = x-\frac{2x}{y}+y-2$

$\Rightarrow \quad -\frac{2x}{y}+y-2=0$ ...(i)

If one student is less in each row, then there are 3 rows more, i.e., when each row has $\left(\frac{x}{y}-1\right)$ students, the number of rows is $(y+3)$.

$\therefore \quad$ Total number of students = No. of rows × No. of students in each row

$\Rightarrow \quad x = \left(\frac{x}{y}-1\right)(y+3)$

$\Rightarrow \quad x = x+\frac{3x}{y}-y-3$

$\Rightarrow \quad \frac{3x}{y}-y-3=0$ ...(ii)

Putting $\frac{x}{y} = u$ in (i) and (ii), we get

$-2u + y - 2 = 0$ ...(iii)

and, $\quad 3u - y - 3 = 0$ ...(iv)

Adding (iii) and (iv), we get

$u - 5 = 0 \quad \Rightarrow \quad u = 5$

Putting $u = 5$ in (iii), we get y = 12

Now, $u = 5 \quad \Rightarrow \quad \frac{x}{y} = 5 \quad \Rightarrow \quad \frac{x}{12} = 5 \quad \Rightarrow \quad x = 60$

Thus, the number of students in the class is 60.

25. Consider two right triangles ABC and PQR such that sinB = sinQ.

We have,

$\sin B = \frac{AC}{AB}$ and $\sin Q = \frac{PR}{PQ}$

$\therefore \quad \sin B = \sin Q$

$\Rightarrow \quad \frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow \quad \frac{AC}{PR} = \frac{AB}{PQ} = k$ (say) ...(i)

$\Rightarrow \quad AC = kPR$ and $AB = kPQ$ ...(ii)

Using Pythagoras theorem in triangles ABC and PQR, we have

$AB^2 = AC^2 + BC^2$ and $PQ^2 = PR^2 + QR^2$

$\Rightarrow \quad BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

$\Rightarrow \quad \frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}}$

$= \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}}$

$= \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k$ ...(iii)

From (i) and (iii), we have

$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \quad \Delta ACB \sim \Delta PRQ$

$\Rightarrow \quad \angle B = \angle Q$

26. The given system of equations may be written as

$(a-b)x + (a+b)y - (a^2 - 2ab - b^2) = 0$

$(a+b)x+(a+b)y-(a^2+b^2)=0$

By cross-multiplication, we have

$$\frac{x}{(a+b)\times(a^2+b^2)-(a+b)\times-(a^2-2ab-b^2)}=\frac{-y}{(a-b)\times-(a^2+b^2)-(a+b)\times-(a^2-2ab-b^2)}=\frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\Rightarrow \quad \frac{x}{-(a+b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}=\frac{-y}{-(a-b)(a^2+b^2)+(a+b)(a^2-2ab-b^2)}=\frac{1}{(a-b)(a+b)-(a+b)^2}$$

$$\Rightarrow \quad \frac{x}{(a+b)\{(a^2+b^2)+(a^2-2ab-b^2)\}}=\frac{-y}{(a+b)(a^2-2ab-b^2)-(a-b)(a^2+b^2)}=\frac{1}{(a+b)(a-b-a-b)}$$

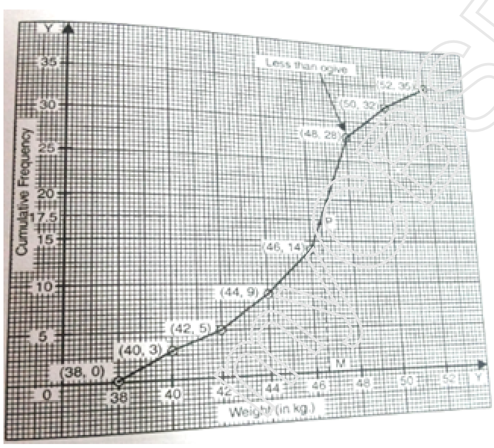
$$\Rightarrow \quad \frac{x}{(a+b)(-2ab-2b^2)}=\frac{-y}{a^3-a^2b-3ab^2-b^3-a^3-ab^2+a^2b+b^3}=\frac{1}{-(a+b)2b}$$

$$\Rightarrow \quad \frac{x}{-2b(a+b)^2}=\frac{-y}{-4ab^2}=\frac{1}{-2b(a+b)}$$

$$\Rightarrow \quad x=\frac{-2b(a+b)^2}{-2b(a+b)}=a+b \text{ and } y=\frac{4ab^2}{-2b(a+b)}=\frac{-2ab}{a+b}$$

Hence, the solution of the given system of equations is $x = a + b$, $y= \frac{-2ab}{a+b}$.

27. To draw the required ogive, we plot the points (38, 0), (40, 3), (44, 9), (46, 14), (48, 28), (50, 32) and (52, 35) and join them by a freehand curve.



To obtain the value of the median, we locate the point $\frac{n}{2}=\frac{35}{2}$ = 17.5 on the y-axis. From this point, we draw a line parallel to the x-axis, meeting the ogive at the point P. From P, we draw a perpendicular PM on the x-axis. The x-coordinate of the point where this perpendicular meets the x-axis, i.e., M gives the value of the median.

$\therefore$ The required value of the median is 46.5 kg.

Verification:

Weight (kg)	Number of students (f_i)	Cumulative frequency (cf)
38-40	3	3
40-42	2	5
42-44	4	9
44-46	5	14
46-48	14	28
48-50	4	32
50-52	3	35

Here, $n = 35$, $\therefore$ $\frac{n}{2} = 17.5$

Median class is 46-48

$\therefore$ $l = 46, f = 14, cf = 14, h = 2$

Median $= l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$

$= 46 + \left(\frac{17.5 - 14}{14}\right) \times 2$

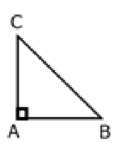
$= 46 + \frac{3.5}{14} \times 2$

= 46.5 kg

The value of the median in both the cases is same, i.e., 46.5 kg.

Hence verified.

28. Draw a right triangle ABC in which $\angle ABC = \theta$



Since, $\cot\theta = \frac{AB}{AC} = \frac{7}{8}$

$\therefore$ Let AB = 7 units and AC = 8 units

$\therefore$ $BC = \sqrt{AB^2 + AC^2}$ (By Pythagoras Theorem)

$= \sqrt{7^2 + 8^2}$

$= \sqrt{49 + 64}$

$= \sqrt{113}$ units

$\therefore \quad \sin\theta = \frac{AC}{BC} = \frac{8}{\sqrt{113}}$ and $\cos\theta = \frac{AB}{BC} = \frac{7}{\sqrt{113}}$

Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{1-\sin^2\theta}{1-\cos^2\theta}$

$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}$

$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$

$= \frac{113-64}{113-49} = \frac{49}{64}$

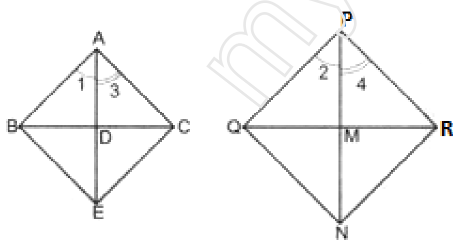
29. Given: In ΔABC and ΔPQR; AD and PM are their medians respectively such that

$\frac{AB}{PQ} = \frac{AD}{PM} = \frac{AC}{PR}$ ... (i)

To Prove: $\Delta ABC \sim \Delta PQR$

Construction: Produce AD to E such that AD = DE and produce PM to N such that PM = MN . Join BE, CE, QN, RN.

Proof: Quadrilateral ABEC and PQNR are $||^{gm}$ because their diagonals bisects each other at D and M respectively.



$\Rightarrow BE = AC$ and $QN = PR$

$\Rightarrow \frac{BE}{QN} = \frac{AC}{PR} \Rightarrow \frac{BE}{QN} = \frac{AB}{PQ}$ [From(i)]

i.e., $\frac{AB}{PQ} = \frac{BE}{QN}$ ...(ii)

From $(i)\dfrac{AB}{PQ}=\dfrac{AD}{PM}=\dfrac{2AD}{APM}=\dfrac{AE}{PN}$

$\Rightarrow \dfrac{AB}{PQ}=\dfrac{AE}{PN}$ ...(iii)

From (ii) and (iii)

$\dfrac{AB}{PQ}=\dfrac{BE}{QN}=\dfrac{AE}{PN}$

$\Rightarrow \Delta ABE \sim \Delta PQN$ (SSS similarity criterion)

$\Rightarrow \angle 1=\angle 2$ ...(iv)

Similarly, we can prove

$\Delta ACE \sim \Delta PRN \Rightarrow \angle 3=\angle 4$ ...(v)

Adding (iv) and (v), we get

$\angle 1+\angle 3=\angle 2+\angle 4 \Rightarrow \angle A=\angle P$

And $\dfrac{AB}{PQ}=\dfrac{AC}{PR}$ (Given)

$\therefore \Delta ABC \sim \Delta PQR$ (By SAS criterion of similarity)

30. If $a = c$, then

$a+\sqrt{b}=c+\sqrt{d} \Rightarrow \sqrt{b}=\sqrt{d} \Rightarrow b = d$

So, let $a\neq c$. Then, there exists a positive rational number x such that $a = c + x$.

Now, $a+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow c+x+\sqrt{b}=c+\sqrt{d}$

$\Rightarrow x+\sqrt{b}=\sqrt{d}$ ...(i)

$\Rightarrow \left(x+\sqrt{b}\right)^2=\left(\sqrt{d}\right)^2$

$\Rightarrow x^2+2\sqrt{b}x+b=d$

$\Rightarrow d-x^2-b=2x\sqrt{b}$

$\Rightarrow \sqrt{b}=\dfrac{d-x^2-b}{2x}$

$\Rightarrow \sqrt{b}$ is rational. $\left[\because d, x \text{ and } b \text{ are rationals}, \therefore \dfrac{d-x^2-b}{2x} \text{ is rational}\right]$

From (i), we have

$\sqrt{d} = x + \sqrt{b}$

$\Rightarrow$ $\sqrt{d}$ is rational

$\Rightarrow$ d is the square of a rational number.

Thus, either $a = c$ and $b = d$ or b and d are the squares of rationals.

31. (a) According to the given condition:

$y = 50 + 25(x - 1)$

$= 50 + 25x - 25$

$\Rightarrow$ $y = 25x + 25$

(b) Correct fare = $25 \times 10 + 25$

$= 250 + 25$

= Rs 275

Amount paid back by the driver = 300 – 275 = Rs 25

(c) The values depicted by the driver in the question are honesty and truthfulness.